Model Based and Empirical Approaches to Robust Control Structure Selection Using the $\mathcal{H}_2$-norm.

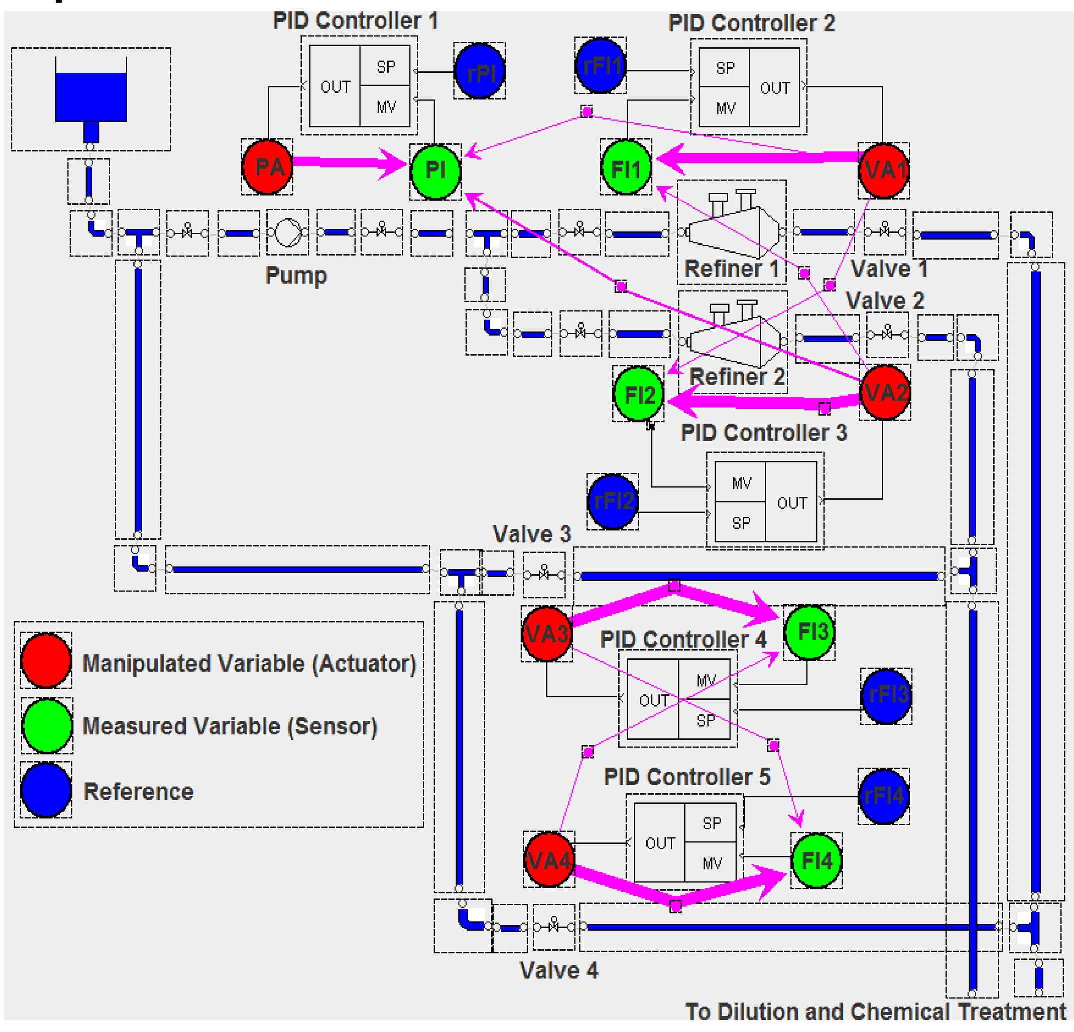
Miguel Castaño Arranz, Wolfgang Birk

LULEÅ UNIVERSITY OF TECHNOLOGY

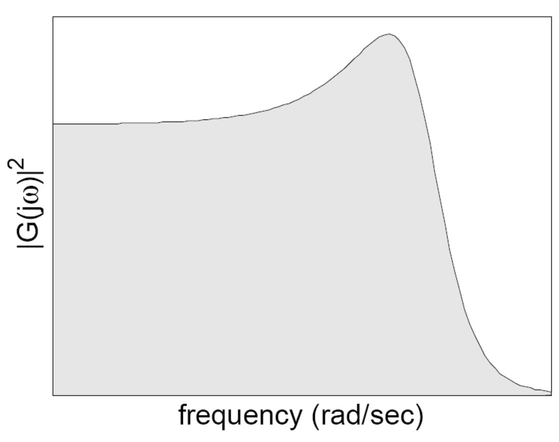
Abstract: The $\mathcal{H}_2$-norm has been successfully used to identify the significant input-output (I/O) interconnections in multivariable systems for the nominal case [1]. We derive in this paper uncertainty regions for the $\mathcal{H}_2$-norm applicable to uncertain parametric models or estimated non-parametric models in the frequency domain, enabling in both cases robust control structure selection.

Control Structure Selection using the $\mathcal{H}_2$-norm.

- Find the I/O channels which are important for process performance.

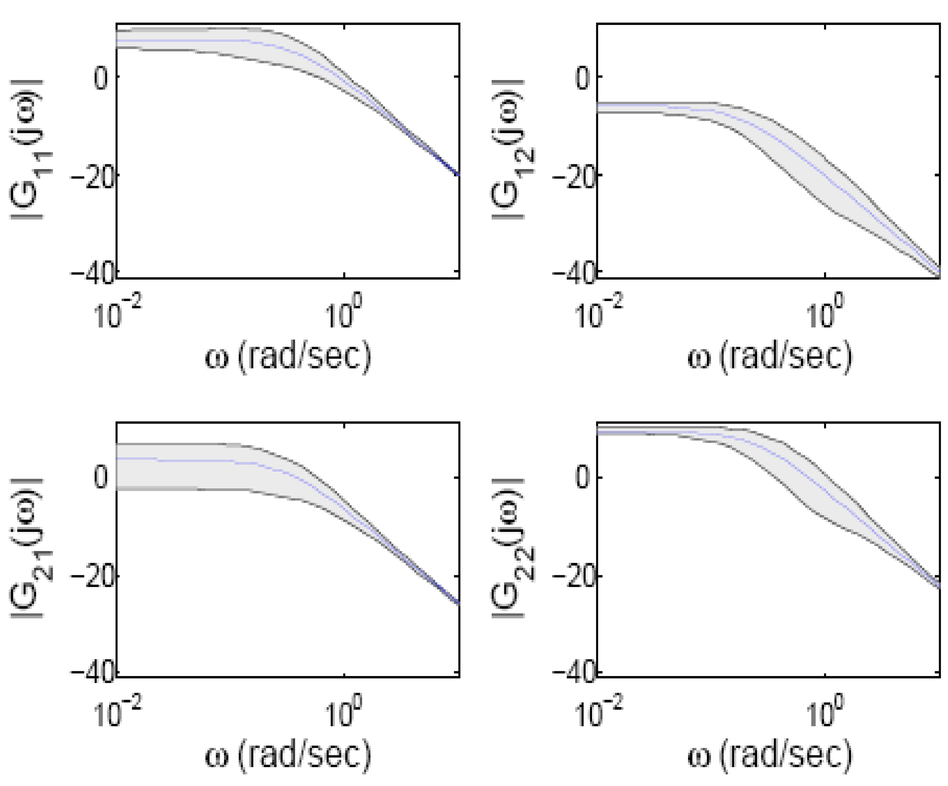


- The $\mathcal{H}_2$-norm can be used to quantify the importance of the I/O channels.
- The $\mathcal{H}_2$-norm is proportional to the area under the squared magnitude of the I/O subsystem.



$$||G(s)||_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} |G(j\omega)|^2 d\omega}$$

- Example: a 2x2 uncertain model.



For the nominal system:

$$[||G_{ij}||_2] = \begin{pmatrix} 1.12 & 0.16 \\ 0.61 & 1.06 \end{pmatrix}$$

- For facilitating the comparison of the I/O channels, the result can be normalized so the sum of all the contributions equals 1. The Interaction Measure named Σ_2 was then defined as [1]:

$$[\Sigma_2]_{ij} = \frac{||G_{ij}||_2}{\sum_{k,l} ||G_{kl}||_2} \quad \Rightarrow \quad \Sigma_2 = \begin{pmatrix} 0.38 & 0.05 \\ 0.21 & 0.36 \end{pmatrix}$$

For the example above

- In the nominal case, the sum of the diagonal elements is 0.74. Which means that a decentralized controller using the diagonal pairing considers 74% of the proces dynamics.
- Control structures which consider at least 70% of the dynamics are likely to derive in acceptable performance.

Robust Control Structure Selection.

- From the uncertain process model previously defined, the uncertainty in the $\mathcal{H}_2$-norm can be computed from the uncertainty in the area under the squared magnitude of the I/O elemental subsystems.

$$[||G_{ij}||_2] \in \begin{pmatrix} [0.82, 1.40], [0.11, 0.21] \\ [0.39, 0.84], [0.77, 1.37] \end{pmatrix}$$

- The uncertainty in the Σ_2 Interaction Measure is:

$$\Sigma_2 \in \begin{pmatrix} [0.26, 0.52], [0.03, 0.09] \\ [0.12, 0.33], [0.24, 0.51] \end{pmatrix}$$

- Considering all the uncertainty set, the previously designed decentralized control structure is unlikely to derive in satisfactory performance (only 60% of the process dynamics might be reflected):

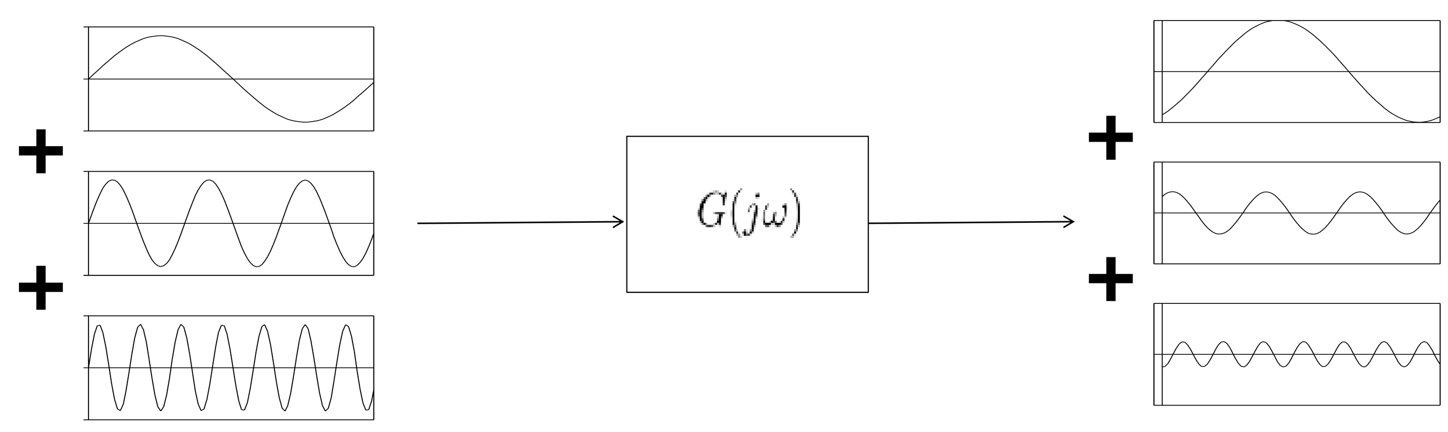
$$[\Sigma_2]_{11} + [\Sigma_2]_{22} \in [0.60, 0.85]$$

- When considering model uncertainty, our decision would be to use a triangular controller, which will consider more than 90% of the process dynamics:

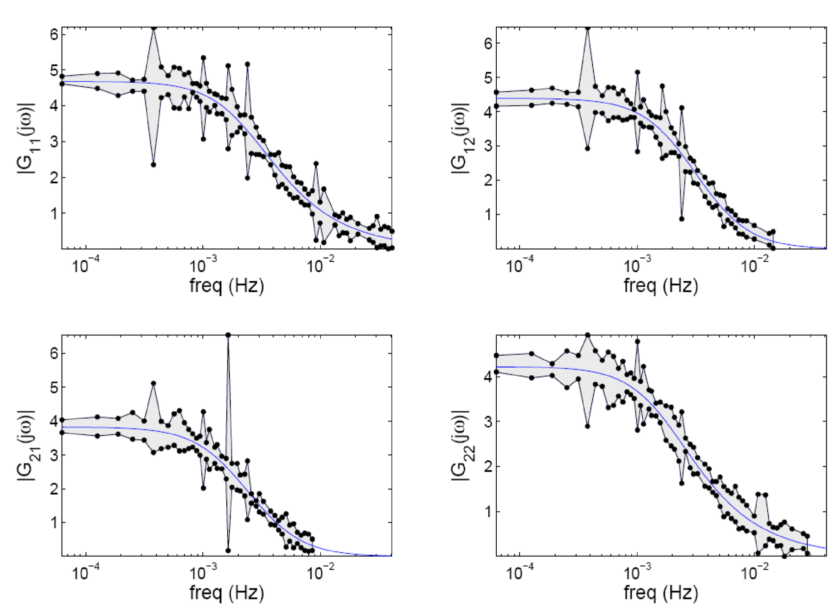
$$[\Sigma_2]_{11} + [\Sigma_2]_{22} + [\Sigma_2]_{21} \in [0.91, 0.97]$$

Estimation of the $\mathcal{H}_2$-norm for Robust Control Structure Selection.

1. Excite your process at multiple frequencies.



2. Estimate the Frequency Response at the excited frequencies.



3. Robust estimation of process interactions, and control structure selection.

$$\Sigma_2 \in \begin{pmatrix} [0.25, 0.39] & [0.19, 0.31] \\ [0.15, 0.25] & [0.19, 0.31] \end{pmatrix}$$

No I/O channel can be discriminated and a full multivariable controller should be used.

Conclusions.

- Considering model uncertainty might derive in a different control structure than the one chosen for the nominal model.
- Estimating Interaction Measures removes the need of process models for the design of control structures.

[1] W. Birk & A. Medvedev, A note on gramian-based Interaction Measures, Proc. of European Control Conference ECC 2003, Cambridge, UK.

Contact: Miguel Castaño (miguel.castano@ltu.se)
Wolfgang Birk (wolfgang.birk@ltu.se)

Funded by: VINNOVA